

terminations in the second order we obtain for this ratio 1.97 as compared with 2.00 predicted by Bohr.

The ratio of the intensities of  $\beta$  to  $\gamma$  in the first order is 6.3 (one determination) and in the second order is 5.46 (two determinations).

<sup>1</sup> Duane and Patterson, *Proc. Nat. Acad. Sci., Washington*, Sept. 1920; *Phys. Rev.*, Dec. 1920, p. 526.

<sup>2</sup> Coster, *Z. Phys.*, **5**, 1921 (139); also **6**, 1921 (185).

<sup>3</sup> De Broglie and Dauvillier, *Paris, C. R.*, **173**, p. 137. Dauvillier, *Ibid.*, **172**, also **173**, p. 647, also other papers in this volume.

<sup>4</sup> Smekal, *Z. Phys.*, **5**, 1921 (91, 121). Also **4**, 1921.

<sup>5</sup> Wentzel, *Ibid.*, **6**, 1921 (2).

<sup>6</sup> Coster, *Ibid.*, **5**, 1921 (147). Also *Physic Rev.*, Sept. 1921.

<sup>7</sup> Duane and Patterson, *Physic Rev.*, **16**, 1920 (534).

<sup>8</sup> Coster, *Ibid.*, Jan. 1922.

<sup>9</sup> Hoyt, *Proc. Nat. Acad. Sci., Washington*, Nov. 1920.

<sup>10</sup> Overn, *Physic. Rev.*, Nov. 1921, p. 350.

## A NEW METHOD OF USING X-RAYS IN CRYSTAL ANALYSIS

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The procedure described in this paper differs in some fundamental details from the elegant methods of analyzing crystals by means of X-rays developed by Laue, the Braggs and others.

The substance to be examined may be in the form of powder or of whole crystals. In the latter case the crystal need not be large. One having a volume of only .25 cubic mm. suffices, although slightly more accurate data may be obtained, if it has a somewhat larger volume.

In our method we employ X-rays belonging to the continuous spectrum, and therefore containing rays of a great variety of wave-lengths, and not the line spectra of definite wave-lengths, as in most other methods. This procedure has certain advantages. An ordinary X-ray tube with a tungsten target may be used, and high voltages may be applied to it. More intense X-radiation can be obtained from tungsten than from such chemical elements as rhodium and molybdenum. Further in some cases it is desirable to employ X-rays of shorter wave-lengths than those in the *K* series of rhodium and molybdenum, for these shorter rays are more penetrating than the longer ones. As a rule we use the spectrum between .12 and .80 Ångström.

In making an analysis of a small crystal we place it at the center of an X-ray spectrometer, provided with an ionization chamber and a quadrant electrometer. The slits of the instrument should be fairly narrow, and

the best results can be obtained by turning the X-ray tube so that the rays that pass through the instrument leave the target almost grazing its surface. This gives a narrow beam of rays. We set the ionization chamber so that the line from the axis of rotation to the chamber's slit makes an angle,  $2\theta$ , of convenient size (usually about  $10^\circ$ ) with the direct beam of rays. Keeping the ionization chamber always in the same position we turn the crystal around by successive small steps and measure the ionization current at each step. When a set of parallel planes in the crystal lie at an angle  $\theta$  to the incident beam of rays the planes pick out of the beam X-rays of certain definite wave-lengths,  $\lambda$ , and reflect them into the chamber, and we get a marked increase in the ionization current. Evidently the angles between the positions of the crystal that correspond to maximum ionization currents are the angles between the crystal planes.

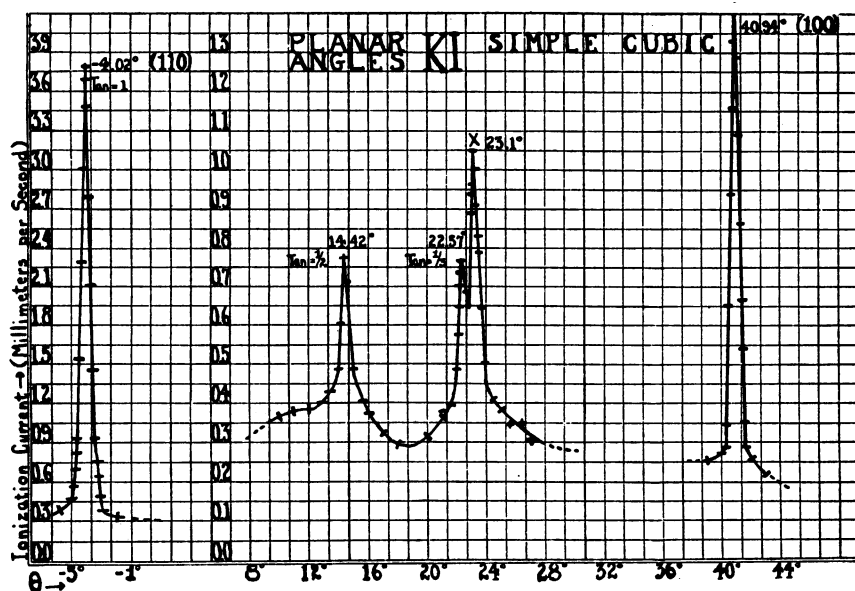


FIGURE 1

The curve in figure 1 represents the ionization current as a function of the position of the crystal in an experiment with potassium iodide. The salt came from Professor Baxter's laboratory, and was of exceptional purity. The peaks on the curve are sharply marked, and their angular positions can be estimated to within less than  $0^\circ.01$ . The angles between several pairs of peaks are  $44^\circ.96$ ,  $18^\circ.37$  and  $26^\circ.52$ , which correspond very closely with the angles between some of the principal planes in a cubic lattice, namely,  $45^\circ$ ,  $18^\circ.43$  and  $26^\circ.57$ . This together with a similar ex-

periment performed with the crystal turned through  $90^\circ$  indicates that *KI* has a cubic lattice, which agrees with previous work by Davey and Wyckoff. The accuracy with which the experimentally determined angles agree with those given by solid geometry amounts in reality to a test of the precision with which the instrument has been set up, i.e., so that an axis of the crystal lies parallel to the axis of rotation, and so that the center of the incident beam of rays and the effective reflecting plane of the crystal lie on the axis of rotation.

One peak of the curve, marked *X*, does not correspond to any of the chief planes of the crystal, i.e., those with small indices. Its position shows that the corresponding planes make an angle of  $17^\circ.84$  with the 100 planes. Possibly the marked reflection at this angle may be due to the characteristic line spectrum of iodine. Further evidence on this point will be given in a later paragraph.

The wave-lengths,  $\lambda$ , of the X-rays reflected at the angle  $\theta$ , corresponding to any one of the peaks, are given by the fundamental law of X-ray reflection, which may be expressed by the equation

$$n\lambda = 2d \sin \theta, \quad (1)$$

where  $n$  is a whole number, and  $d$  is the distance between successive planes. We cannot use this equation alone to determine  $d$ , for we do not yet know the value of  $\lambda$ . The peaks on the curve in figure 1 give us the angles between various planes only.

In order to determine the distance,  $d$ , we make use of the fact<sup>1</sup> that, in order to produce an X-ray of wave-length  $\gamma$ , the voltage,  $V$ , applied to the X-ray tube must have a value at least as large as that given by the equation

$$Ve = \frac{hc}{\lambda}, \quad (2)$$

where  $h$  is Planck's radiation constant,  $e$  the charge of an electron ( $e$ ) and  $c$  the velocity of light. This equation may be written in the form

$$V\lambda = \frac{hc}{e} = 12,354, \quad (2')$$

where the numerical value<sup>2</sup> of the universal constant,  $hc/e$ , corresponds to  $V$  expressed in volts and  $\gamma$  in Ångströms.

In an actual experiment to measure  $d$  we set the crystal so that the X-rays reflected by the planes we desire to investigate enter the ionization chamber (i.e., we set on one of the peaks in the curve of figure 1), and reduce the voltage until the ionization current sensibly vanishes. The voltage at which it vanishes is sharply marked, especially if the voltage is a constant one, from a storage battery, for instance. It is a good plan to increase the current through the tube at the same time as the voltage decreases in such a way as to keep the product of the two, that is, the

power delivered to the tube, constant. Evidently the ionization current will not completely vanish until the voltage becomes so low that it cannot produce the *longest* X-rays reflected at the angle  $\theta$  according to equation (1). The longest waves correspond to  $n = 1$ .

A small correction has to be added to the angle  $\theta$  due to the fact that the source of rays and the slit of the spectrometer have finite breadths. The rays in the incident beam are not all exactly parallel to each other and the correction equals half the angle between the rays that deviate most from the central ray on its two sides. This angle is the breadth of the source plus the breadth of the slit divided by the distance between them. The correction is usually small. In our experiments it amounts to about  $3'$  of arc, which is less than one per cent of the corrected value of  $\theta = 5^\circ.51$  used in the first illustrative case described below.

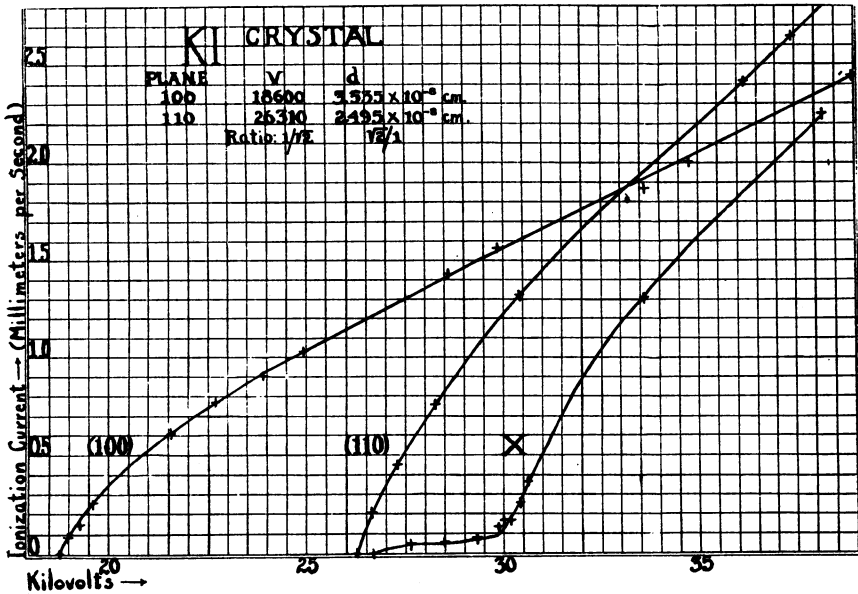


FIGURE 2

The curves in figure 2 represent the ionization currents as functions of the voltages for three of the peaks of figure 1. The voltages at which the ionization currents vanish for the peaks marked 100 and 110, respectively, are 18,600 and 26,310 volts.

Eliminating  $\lambda$  between equations (1) and (2') we get, with  $n = 1$ ,

$$d = \frac{12,354}{2V \sin \theta} \tag{3}$$

Substituting in this  $\theta = 5^\circ.51$  and the values of  $V$ , we find for  $d$  the 3.535 and 2.495 Ångströms, respectively. The ratio of the first to the

second is 1.417, which differs little from  $\sqrt{2} = 1.414$ , the value it should have in a cubic crystal. Taking 3.532 as the average length of the edge of a cube, and assuming one atom to each cube the calculated density of KI comes out 3.111, in close agreement with the density 3.114 measured by Baxter.

Equation (3) contains a universal constant  $\frac{hc}{e} = 12,354$  and two quantities, a voltage,  $V$ , and an angle,  $\theta$ , both of which we measure. The values of  $d$ , therefore, determined by this method are independent of previous measurements of the distance between the planes in any crystal. This advantage is, perhaps, more theoretical than practical at present, for measurements with X-rays reflected from calcite probably furnish us the most accurate estimates we have for the universal constant.

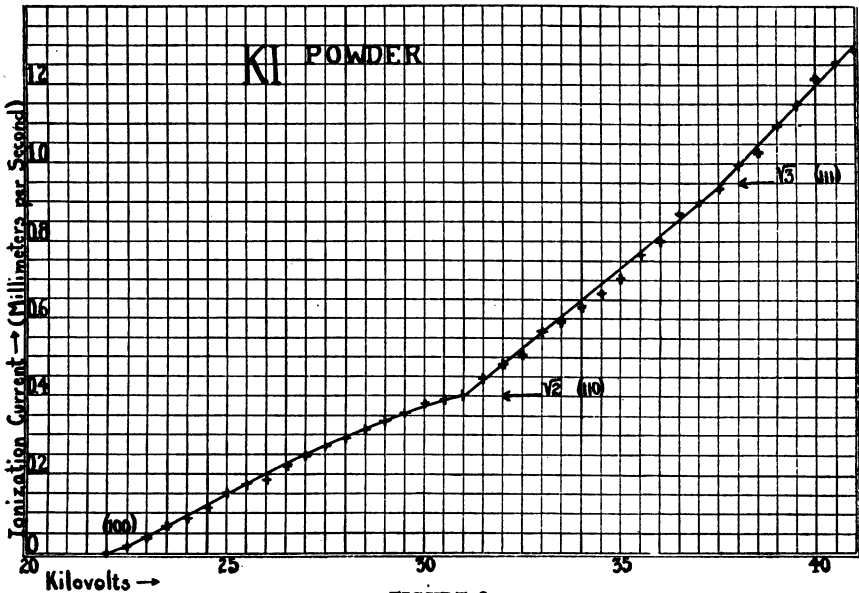


FIGURE 3

The agreement between the distances,  $d$ , determined by this method with those calculated from the density, atomic weights, etc., helps to confirm the general correctness of the laws of X-rays as at present understood.

If the specimen to be examined has the form of powder, we place the powder at the center of the spectrometer, fix the position of the ionization chamber at an angle  $2\theta$  and reduce the voltage by steps making measurements of the ionization current as before. The ionization current vanishes when the voltage is no longer able to produce the longest X-rays that are reflected at the angle  $\theta$  by the powder. This maximum wave-length corresponds to  $n = 1$  in equation (1) and to the greatest distance  $d$  be-

tween successive planes in the crystal that are alike. The curve in figure 3 represents the ionization current as a function of the voltage in an experiment with potassium iodide powder. In addition to the point where the ionization current vanishes there are several well-marked places where the curvature changes abruptly. The voltages at these breaks in the curve are the voltages at which the rays reflected from other crystal planes cease to be produced. These voltages together with the angle  $\theta$ , substituted in equation (3), give the corresponding distances between the planes. The three most sharply determined voltages are 21,740, 31,010 and 37,600 volts, respectively. The ratios of these numbers to each other differ from  $\sqrt{2}$  and  $\sqrt{3}$  by small fractions of one per cent only. They correspond, therefore, to the 100, 110 and 111 planes, respectively, of a cubic lattice. The angle  $\theta$  equals  $4^{\circ}.51$  in this experiment, and the average value of  $d$ , calculated from equation (3), is 3.534, in close agreement with that obtained in the above-described measurement with a small crystal of potassium iodide.

The curve marked  $X$  in figure 2 corresponds to the peak marked  $X$  in figure 1. There appears to be a sharply indicated point of inflection in the former at a voltage of 30,000 volts. Substituting this in equation (2') we find that the corresponding wave-length is .412. This wave-length is a good mean value of the wave-lengths of  $\alpha$  and  $\beta$  lines in the  $K$  series of iodine, namely, .437 and .388. 30,000 volts do not suffice to produce the  $K$  series of iodine. If, however, one sets a spectrometer on a characteristic line and reduces the voltage, the ionization current does not vanish when the voltage ceases to produce the characteristic X-rays. It vanishes only when the voltage becomes insufficient to produce an X-ray in the continuous spectrum of equal wave-length to that of the characteristic line. We have tested the existence of the peak  $X$  by turning the crystal as in the experiments represented in figure 1 with voltages in the neighborhood of 33,030 volts, which is the critical voltage for the production of the  $K$  series of iodine. We find no evidence of a peak at a voltage slightly below 33,000. The peak is just observable, however, if the voltage equals 33,500 volts. These points strongly favor the idea that the characteristic  $K$  radiation of iodine produces the peak  $X$ . The distance between the planes making an angle of  $17^{\circ}.84$  with the 100 planes, however, does not equal that calculated from equations (1) and (3). The former is 1.088 and the latter is 2.144, i.e., about twice the former. Perhaps this may be due to the fact that only half the atoms are iodine. If the characteristic radiation does produce the peak, and, if we adopt the interference of scattered rays as the explanation of X-ray reflection, we must assume that, on the average at least, definite time relations must exist between the passage of the primary X-ray across an atom and the emission of a secondary *characteristic* ray. If we adopt the quantum point

of view, we must suppose that the direction in which the characteristic ray travels depends upon neighboring atoms as well as upon the angle of incidence of the primary beam.

The relative reflecting power of different planes may be determined by reflecting from them the *same* portion of the continuous spectrum, or different portions having known relative intensities. The use of the continuous spectrum for this purpose instead of the line spectrum has the advantage of flexibility. One can employ short, penetrating X-rays, and with these investigate dense crystals of high absorbing power.

<sup>1</sup> *Phys. Rev., Ithaca*, Aug. 1915, p. 166.

<sup>2</sup> *J. Optical Soc.*, May 1921, p. 386.

## ACOUSTIC TOPOGRAPHY VARYING WITH THE POSITION OF THE ORGAN PIPE\*

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1. *Introductory.*—The coordinate axes have the  $x$ - $y$  plane on the large table, the origin near one corner. The survey is made between walls at  $y = 174$  cm. and  $y = -130$  cm.; and the pipe is displaced both in the  $y$  and  $z$  (upward) direction. In the graphs, the letters  $P$ ,  $R$ ,  $T$ ,  $T'$ , used either as subscripts or directly, will refer to the pipe, the resonator, the edges of the table. Throughout  $s$  is the fringe deflection of the U-tube interferometer, proportional to the acoustic pressure or to the nodal density evoked at a given place ( $x = 0$ ,  $y$ ,  $z$ ), by the sounding pipe  $P$ .

The latter was a closed brass  $f''$  pipe, blown by the pipe blower elsewhere described, rigidly attached. In graphs 1-3, this is sketched in under  $P$  and an arrow shows the direction of the wind current. The closed cylindrical pinhole resonator  $R$ , lying with its axis parallel to  $y$ , in the  $yx$  plane (table) and moving end-on along  $y$ , is also suggestively indicated, showing the resonator azimuth for which the usually paired graphs apply. In the earlier work (figures 1-3) the *middle* of the resonator was used for the location of points in  $y$ . An inversion of the resonator thus gave the two curves for each case and they were obtained consecutively. If the mouth of  $R$  is used to determine the coordinate  $y$  as in figures 4, 6 to 10, but a single curve is obtained, for all azimuths of  $R$ . The same is true for a resonator placed with its mouth at the mean  $y_R$  of a given ordinate,  $s$ , in case of figures 1-3.

2. *Pipe Elevated above the Origin on the Table.*—In figures 1-4 the organ pipe, with its mouth vertically above the origin, was raised succes-